Covariate-informed latent interaction models

Addressing geographic & taxonomic bias in predicting bird-plant interactions

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Motivation

- Measured networks are often
  $\rightsquigarrow$ incomplete
    we observed some interactions and recorded them
  $\rightsquigarrow$ measured with error
    incomplete + recorded interactions might not be truly present

- Incomplete measured networks might occur because researchers
  $\rightsquigarrow$ target specific individuals
  $\rightsquigarrow$ have access to only a subset of nodes

- Interest:
  $\rightsquigarrow$ Infer the true interaction network from limited measured networks
  $\rightsquigarrow$ Understand the covariates that drive node interaction

Modeling incomplete networks

- Not necessarily a problem:
  ⇝ If our inferential interest is the population we followed
- Could be a problem:
  ⇝ If the population we want to learn about is dissimilar than the one followed
  ⇝ measured interactions are not representative of interactions among target population

Bipartite interaction networks in ecology

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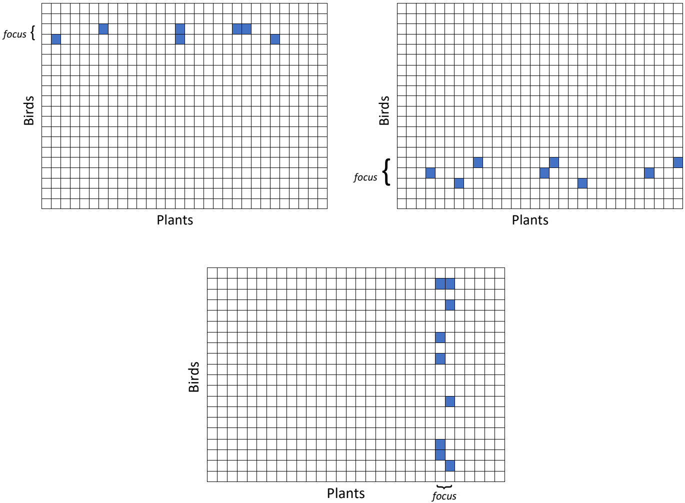

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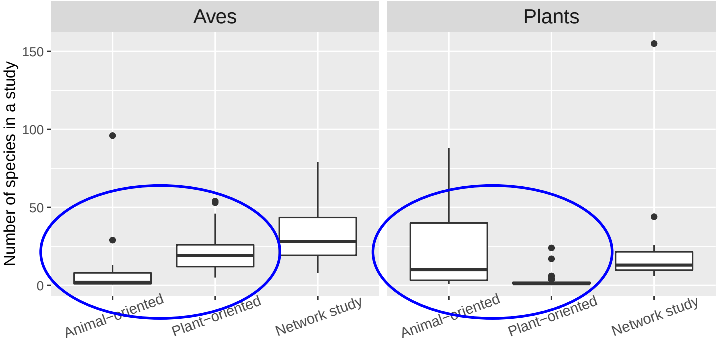

Number of unique species by study type

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- Network studies are most useful for studying species interactions

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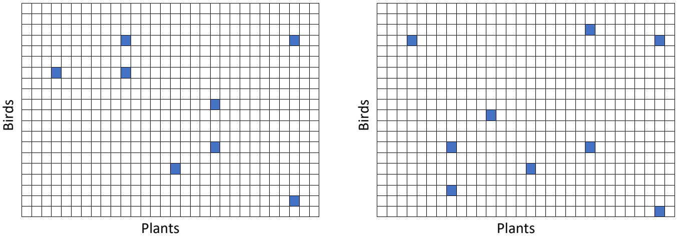

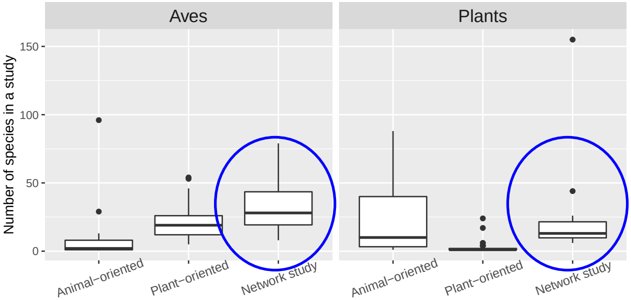

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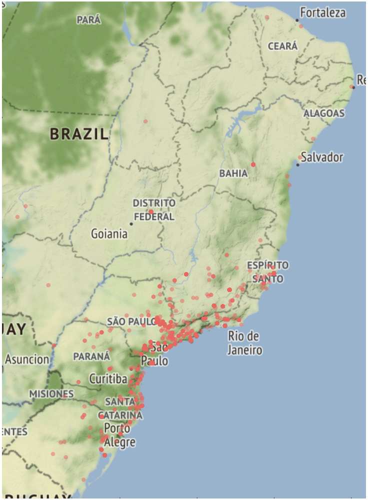

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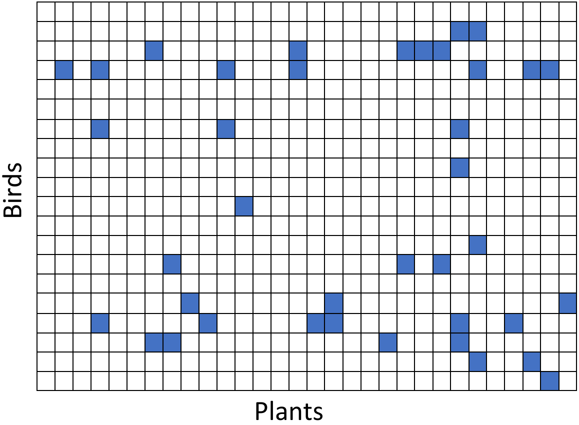

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- Under or over-representation of species
- Combined network is taxonomically and geographically biased

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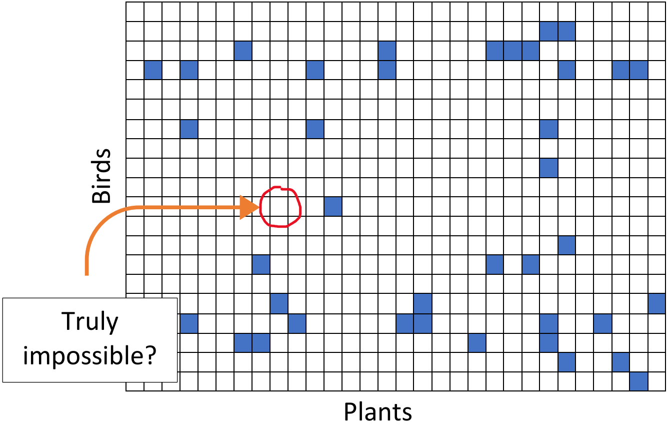

Truly impossible?

Bipartite interaction networks in ecology

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Goals:

1. Understand species interactivity while "adjusting" for these biases
2. Learn which covariates are most important in driving species interactions & detectability

Motivation

- The Atlantic Forest currently includes only 12% of its original biome
- Plants rely on frugivore populations for seed dispersal
- Reductions in frugivore populations lead to disruptions in the regeneration of ecosystems
- Climate change, reductions in natural habitats, deforestation
- How will biological communities respond?

Goals:

- Understand species interactivity
- Would a given bird consume the seed of a given plant, if given the opportunity?
- What are the drivers of species interactions?

Our setup

- $S = 85$ individual studies
  $\rightsquigarrow$ 19 animal-oriented, 45 plant-oriented, and 19 network studies
- bird $i = 1, 2, \ldots, n_B$ $\qquad (n_B = 232)$
- plant $j = 1, 2, \ldots n_P$ $\qquad (n_P = 511)$
- $A_{ijs} = 1$ or 0: recorded or unrecorded interaction in study s
- $\boldsymbol{X}_i, \boldsymbol{W}_j$: covariate information

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Goals (in statistical terms):

- Learn bipartite network of possible interactions
  $\rightsquigarrow$ $L_{ij} = 1$ if interaction is possible, 0 otherwise
  $\rightsquigarrow$ unrecorded interactions are not necessarily impossible
  $\rightsquigarrow$ recorded networks are prone to biases
- Study covariate importance in latent network models

Our Approach

- Elucidate likelihood for $(\boldsymbol{A}, \boldsymbol{X}, \boldsymbol{W})$

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- The measured covariates might not include all relevant information

  Introduce latent factors:

  $\boldsymbol{U}_i = (U_{i1}, \ldots, U_{iH})^T$ for bird species
  $\boldsymbol{V}_j = (V_{j1}, \ldots, V_{jH})^T$ for plant species

  $\rightsquigarrow$ Representation of species covariate information
  $\rightsquigarrow$ Arbitrarily close to species' measured covariates

Our Approach

- Elucidate likelihood for $(\boldsymbol{A}, \boldsymbol{X}, \boldsymbol{W})$
- For a measured network to have recorded a given interaction, all of the following need to happen:
    - species co-occur
    - researchers are interested in the pair of species
    - species truly interact
    - an interaction was detected

Our Approach

- Elucidate likelihood for $(\boldsymbol{A}, \boldsymbol{X}, \boldsymbol{W})$
- Species occurrence: $O_{ijs} = 1$ if i, j both occur at the study site

  $\rightsquigarrow$ important for addressing geographical bias

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- True interactions: $L_{ijs} = 1$ if ij-pair is possible to interact
- Species' detectability: p_i, q_j for bird i and plant j

Our Approach – Dependencies

- Focus on

$$P\big(\boldsymbol{A} = \boldsymbol{a} \mid \boldsymbol{L}, \boldsymbol{F}, \boldsymbol{O}, \{p\}, \{q\}, \{\boldsymbol{U}\}, \{\boldsymbol{V}\}, \{\boldsymbol{X}\}, \{\boldsymbol{W}\}\big)$$

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- Dependencies across measured networks

  $\rightsquigarrow$ geographic proximity (O_{ijs})
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- and specify:

$$P(A_{ijs} = 1 \mid L_{ij} = l, F_{ijs} = f, O_{ijs} = o, p_i, q_j) = \begin{cases} 0, & \text{if} \quad lfo = 0 \\ p_i q_j, & \text{if} \quad lfo = 1 \end{cases}$$

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$$= \prod_{\substack{i,j,s \\ F_{ijs}O_{ijs}L_{ij}=1}} (p_i q_j)^{a_{ijs}} (1 - p_i q_j)^{1-a_{ijs}} \prod_{\substack{i,j,s \\ F_{ijs}O_{ijs}L_{ij}=0}} I\left(a_{ijs} = 0\right)$$

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- Need to specify joint distribution on unobserved quantities:

$$\boldsymbol{L}, \{\boldsymbol{U}\}, \{\boldsymbol{V}\}, \{p\}, \{q\}$$

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$\rightsquigarrow$ Recorded interactions depend on species' characteristics:

$$\mathrm{logit} P(L_{ij} = 1 \mid \boldsymbol{X}_i, \boldsymbol{U}_i, \boldsymbol{W}_j, \boldsymbol{V}_j) = \lambda_0 + \sum_{h=1}^{H} \lambda_h U_{ih} V_{jh}$$

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⇝ Species' detectability depends on species' characteristics:

$$E[\mathsf{logit}(p_i) \mid \boldsymbol{U}_i, \boldsymbol{X}_i] = \delta_0 + \boldsymbol{U}_i^T \boldsymbol{\delta}$$

$$E[\mathsf{logit}(q_j) \mid \boldsymbol{V}_j, \boldsymbol{W}_j] = \zeta_0 + \boldsymbol{V}_j^T \boldsymbol{\zeta}$$

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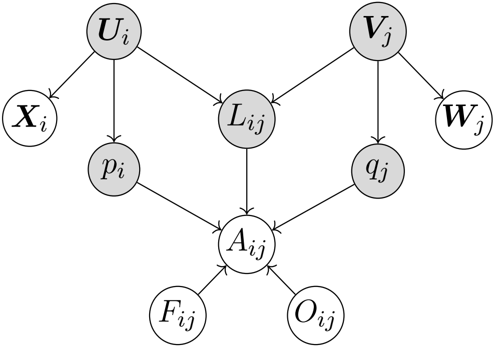
⤳ Latent factors are "close" enough to measured covariates

$$f_m^{-1}(E(X_{im} \mid \boldsymbol{U}_i)) = \beta_{m0} + \boldsymbol{U}_i' \boldsymbol{\beta}_m, \ m = 1, 2, \ldots, p_B, \text{ and}$$
$$g_l^{-1}(E(W_{jl} \mid \boldsymbol{V}_j)) = \gamma_{l0} + \boldsymbol{V}_j' \boldsymbol{\gamma}_l, \ l = 1, 2, \ldots, p_P$$

Our approach (schematically)



Our Approach (prior distributions)

- $\boldsymbol{U}_{.h} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_U)$, and $\boldsymbol{V}_{.h} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_V)$
  - $\rightsquigarrow$ independently across h
  - $\rightsquigarrow$ $\boldsymbol{\Sigma}_U, \boldsymbol{\Sigma}_V$ phylogenetically structured across species

Sirio Legramanti, Daniele Durante, and David B. Dunson. Bayesian cumulative shrinkage for infinite factorizations. Biometrika, 107(3): 745â752, 2020

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- Model coefficients: $\beta_{mh} | \tau^{\beta}_{mh}, \theta_h \sim N(0, \tau^{\beta}_{mh} \theta_h)$ *similarly for others*

  θ_h: The increasing shrinkage prior of Legramanti et al. (2020)

  $\rightsquigarrow$ Increasingly penalizes coefficients with larger h

  τ^{β}_{mh}: Coefficient-specific additional variation

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Computation

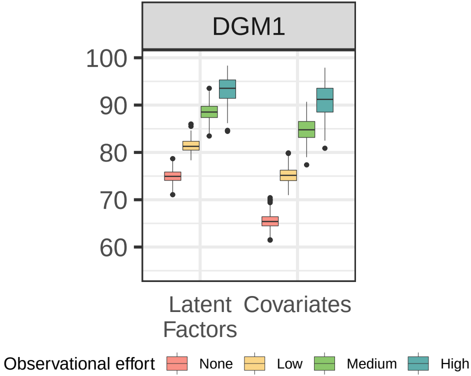
- We approximated the posterior distribution using MCMC
- Most updates were performed using Gibbs / MH
- Pólya-Gamma data augmentation scheme for parameters of logistic models

Simulations

- We considered 24 scenarios:
  - Same or different covariates important for interactions and detectability
  - Important covariates are measured, mixed or unmeasured
  - Covariates are correlated or not
  - High and low information scenarios
- Alternative approaches using covariates, ignoring biases, fixed latent factor dimension ...

Simulations

AUROC – predicting missing interactions



Variable importance in latent network models

- Interaction model does not include covariates
- We cannot interpret coefficients

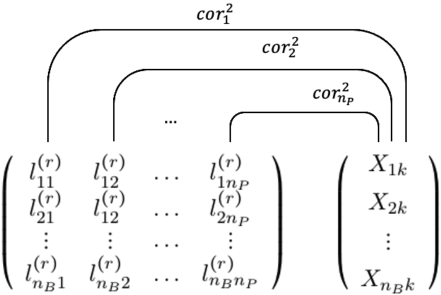
  $\rightsquigarrow$ lack of identifiability of latent factors
- Covariates are not included in the interaction model

  $\rightsquigarrow$ by design

  $\rightsquigarrow$ interpreting coefficients in models with structured latent factors has challenges (Van Ee et al., 2021)

Justin J Van Ee, Jacob S Ivan, Colorado Parks, Wildlife Mevin, B Hooten, and Mevin B Hooten. Community Confounding In Joint Species Distribution Models. 2021.

Variable importance in latent network models

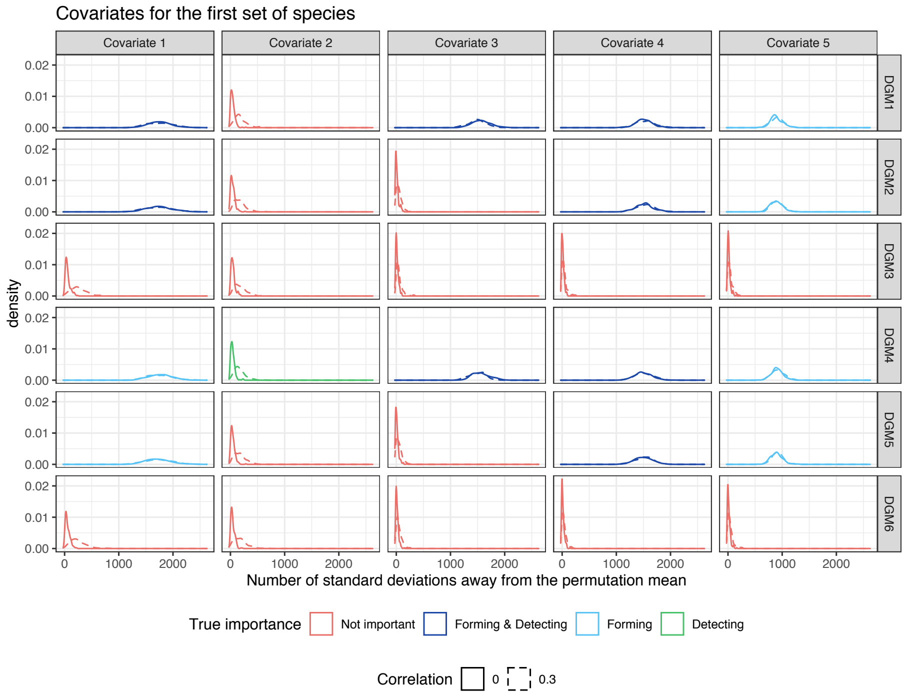

cor_1^2
cor_2^2
$cor_{n_P}^2$
...
$\begin{pmatrix} l_{11}^{(r)} & l_{12}^{(r)} & \dots & l_{1n_P}^{(r)} \\ l_{21}^{(r)} & l_{12}^{(r)} & \dots & l_{2n_P}^{(r)} \\ \vdots & \vdots & \dots & \vdots \\ l_{n_B1}^{(r)} & l_{n_B2}^{(r)} & \dots & l_{n_Bn_P}^{(r)} \end{pmatrix}$
$\begin{pmatrix} X_{1k} \\ X_{2k} \\ \vdots \\ X_{n_Bk} \end{pmatrix}$

$l_{ij}^{(r)}$: r^{th} posterior sample of (logit) probability of interaction between bird i and plant j

1. Calculate T^{obs} by averaging across species and posterior samples
2. Permute the covariate vector B times $\rightsquigarrow T^{(b)}$, $b = 1, 2, \dots, B$
3. Use

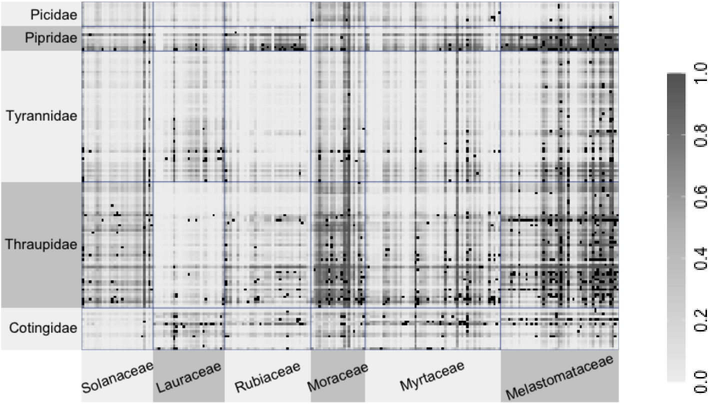
$$T^{obs} - \text{avg}\left(T^{(b)}\right) \quad \Big/ \quad \text{sd}\left(T^{(b)}\right)$$

as the variable importance metric



Covariates for the first set of species

In our study...



Our approach

In our study...

- Species' interaction profiles appear to be taxonomically structured
- 5% of pairs are predicted to interact (post. prob. $> 80\%$)
- 41% of pairs are predicted to *not* interact (post. prob. $< 10\%$)

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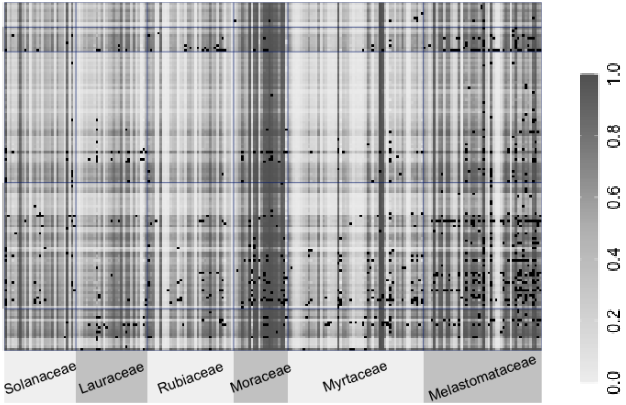


Alternative approach using covariates directly

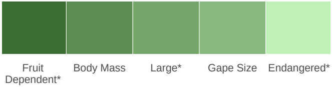
In our study...

- Species' interaction profiles appear to be taxonomically structured
- 5% of pairs are predicted to interact (post. prob. $> 80\%$)
- 41% of pairs are predicted to *not* interact (post. prob. $< 10\%$)
- Cross validation:

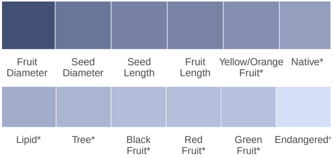
  $\rightsquigarrow$ compare post. prob. of interaction in held-out pairs compared to all pairs

  $\rightsquigarrow$ covariates: (post.prob. interacting) 1.4 times higher (all)
  $\rightsquigarrow$ latent factors: (post.prob. interacting) 3.2 times higher (all)
- Latent factor model differentiates truly possible interactions better

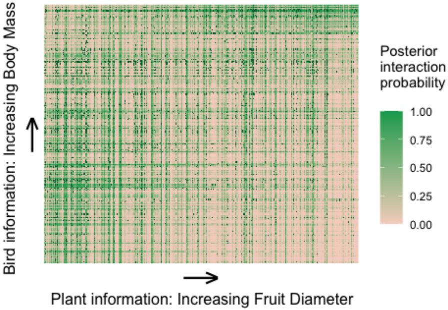
Variable importance on species interactions



(a) Bird Traits Importance



(b) Plant Traits Importance



(c) Interaction matrix ordered by traits

Final thoughts

- Latent network models for noisy bipartite networks

  ⇝ covariates inform the latent factors via separate models
  ⇝ quantifies our uncertainty around the estimated graph
  ⇝ posterior samples + permutation for variable importance

- Study species interactions based on meta-analysis data set

  ⇝ complete the bipartite graph of species interdependence
  ⇝ incorporates the missingness mechanism caused by the taxonomic and geographic bias of individual studies

- EXTENSION: simultaneous modeling of co-occurrence and interactions

  ⇝ incorporate geographic information and environmental variables
  ⇝ investigate the importance of species abundance and competition

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Preprint:	arXiv:2103.05557
Rpackage:	`https://github.com/gpapadog/BiasedNetwork`
Analysis:	`https://github.com/gpapadog/Bird_Plant_Interactions`

Thank you!