Mitigating Unobserved Spatial Confounding Bias with Mixed Models

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Causal inference and unmeasured structured confounding

- Causal inference formalizes the notion of an *effect*, and provides identifiability assumptions
- One often invoked assumption is the no unmeasured confounding assumption (+ positivity = ignorability)
- No unmeasured confounding cannot be tested but sensitivity of results to violations of this assumption can be evaluated [Rosenbaum, 2002]

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- Can we use unmeasured confounders' *structure* to adjust for them?
  - Spatial structure: spatial variables vary continuously over space

Unmeasured spatial confounding in the literature

- Causal literature [Keele et al., 2015, Papadogeorgou et al., 2018]
  - Spatial information in treatment assignment
- Spatial literature [Hodges and Reich, 2010, Paciorek, 2010]
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  → *Confusion about what these spatial models are capable of accounting for*

  → *Spatial random effects do not eliminate bias*
- Spatial and causal inference literatures remain largely separated
- Bridging the two strands of literature by
  - Unmeasured confounding within the causal inference framework
  - Estimation using models and tools common among spatial statisticians

Patrick Schnell

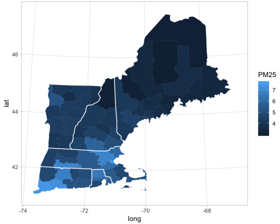
Spatial data and causal inference in air pollution research

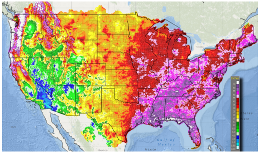
- The scientific questions are causal
  - Do emissions cause pollution?
  - What effect does an intervention on polluting sources have on air pollution concentrations?

https://kcstormfront.wordpress.com/2015/01/11/2014-in-review/

Spatial data and causal inference in air pollution research

- The scientific questions are causal
  - Do emissions cause pollution?
  - What effect does an intervention on polluting sources have on air pollution concentrations?
- The data are spatial
  - Spatially-indexed
  - Exposure, outcome, and covariates are spatially structured
  - Unmeasured confounders are spatial!





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Notation

- For unit i
  - Treatment or exposure $Z_i \in \mathcal{Z}$
  - Potential outcomes $\{Y_i(z), z \in \mathcal{Z}\}$
  - Observed outcome $Y_i = Y_i(Z_i)$
  - Covariates $\boldsymbol{W}_i = (W_{i1}, W_{i2}, \ldots, W_{ip})$
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- Common identifiability assumptions
  - Positivity: $p(Z = z|\boldsymbol{W}) > 0, z \in \mathcal{Z}$
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- Estimate the average potential outcome via propensity score methods, outcome regression, or combinations
- Confounders $\boldsymbol{W} = (\boldsymbol{W}^m, \boldsymbol{W}^u)$, $\boldsymbol{W}^m$ are observed, $\boldsymbol{W}^u$ are unobserved
- If $\boldsymbol{W}^u$ vary spatially, can we adjust for it?

Potential outcomes

- We assume the following *true* model for the potential outcomes:

$$Y_i(z) = \eta(z, \boldsymbol{W}^m) + g(\boldsymbol{W}^u) + \varepsilon_i$$

  - $\boldsymbol{W}^u$ are unmeasured variables, denote $U = g(\boldsymbol{W}^u)$
  - g is such that $E[g(\boldsymbol{W}^u)] = E[U] = 0$
  - Additive model, $\boldsymbol{W}^u$ do not interact with Z and $\boldsymbol{W}^m$
- For ease of presentation, assume $\boldsymbol{W}^m$ empty, $\eta(z) = \beta_0 + \beta_1 z$
- Focus on $\beta_1 = \overline{Y}(z+1) - \overline{Y}(z)$

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- $Y \sim Z +$ Spatial RE $\to \widetilde{\boldsymbol{\beta}}$
  - $\mathrm{E}\big(\widetilde{\boldsymbol{\beta}}|\boldsymbol{Z}\big) = \boldsymbol{\beta} + \{\mathbf{X}^\intercal(\mathrm{Var}[\boldsymbol{Y}|\boldsymbol{Z}])^{-1}\mathbf{X}\}^{-1}\mathbf{X}^\intercal(\mathrm{Var}[\boldsymbol{Y}|\boldsymbol{Z}])^{-1}\mathrm{E}[\boldsymbol{U}|\boldsymbol{Z}]$

    where $\mathbf{X} = (\mathbf{1}, \boldsymbol{Z})$

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    where $\mathbf{X} = (\mathbf{1}, \boldsymbol{Z})$
- Identify the bias term, and subtract it

$$\bar{\boldsymbol{\beta}} = \{\mathbf{X}^\intercal(\mathrm{Var}[\boldsymbol{Y}|\boldsymbol{Z}])^{-1}\mathbf{X}\}^{-1}\mathbf{X}^\intercal(\mathrm{Var}[\boldsymbol{Y}|\boldsymbol{Z}])^{-1}\{\boldsymbol{Y} - \mathrm{E}[\boldsymbol{U}|\boldsymbol{Z}]\}$$

- Find a way to identify $E[\boldsymbol{U}|\boldsymbol{Z}]$!

A Gaussian Markov random field construction of the joint distribution

- Assumptions on the joint distribution of $(\boldsymbol{U}, \boldsymbol{Z})$ to identify $E[\boldsymbol{U}|\boldsymbol{Z}]$
- $(\boldsymbol{U}, \boldsymbol{Z})$ is mean 0 normal

1. **Cross-Markov property:** $p(Z_i|\boldsymbol{Z}_{-i}, \boldsymbol{U}) = p(Z_i|\boldsymbol{Z}_{-i}, U_i)$,
2. **Constant conditional correlation:** $\mathrm{Cor}(U_i, Z_i|\boldsymbol{U}_{-i}, \boldsymbol{Z}_{-i}) = \rho$.

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$$\begin{pmatrix} \boldsymbol{U} \\ \boldsymbol{Z} \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}, \begin{pmatrix} \mathbf{G} & \mathbf{Q} \\ \mathbf{Q}^\intercal & \mathbf{H} \end{pmatrix}^{-1}\right],$$

- $\mathbf{Q}$ is diagonal, and $q_{ii} = -\rho\sqrt{g_{ii}h_{ii}}$
- For areal data, we specify $\mathbf{G}, \mathbf{H}$ as CAR

Calculating the affine estimator

- Integrating $\mathbf{U}|\boldsymbol{Z}$ out

$$\boldsymbol{Y}|\boldsymbol{Z} \sim \mathcal{N}[\mathbf{X}\boldsymbol{\beta} - \mathbf{G}^{-1}\mathbf{Q}\boldsymbol{Z}, \mathbf{G}^{-1} + \mathbf{R}^{-1}],$$
$$\boldsymbol{Z} \sim \mathcal{N}[\mathbf{0}, (\mathbf{H} - \mathbf{Q}^\intercal\mathbf{G}^{-1}\mathbf{Q})^{-1}]$$

  where $\mathbf{R}^{-1} = \mathrm{Cov}(\boldsymbol{\varepsilon})$

- Parameters are estimated based on the restricted likelihood

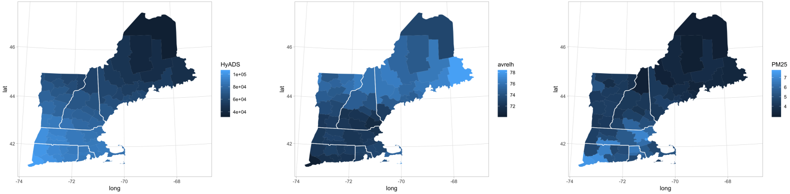
$$RL \propto C_1 \exp\left[-\frac{1}{2}\left\{(\boldsymbol{Y} - \mathbf{B}\boldsymbol{Z})^\intercal C_2(\boldsymbol{Y} - \mathbf{B}\boldsymbol{Z}) + \boldsymbol{Z}^\intercal\mathbf{A}^{-1}\boldsymbol{Z}\right\}\right]$$

  where $\mathbf{A} = (\mathbf{H} - \mathbf{Q}^\intercal\mathbf{G}^{-1}\mathbf{Q})^{-1}$, and $\mathbf{B} = -\mathbf{G}^{-1}\mathbf{Q}$

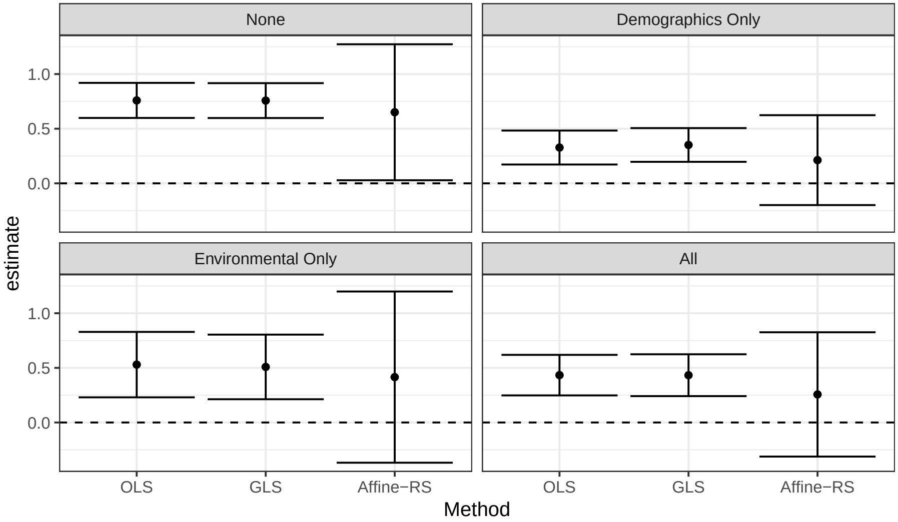
- *Spatial scale restriction* [Paciorek, 2010]
- We calculate $\bar{\boldsymbol{\beta}}$ using the RL maximizers

Data

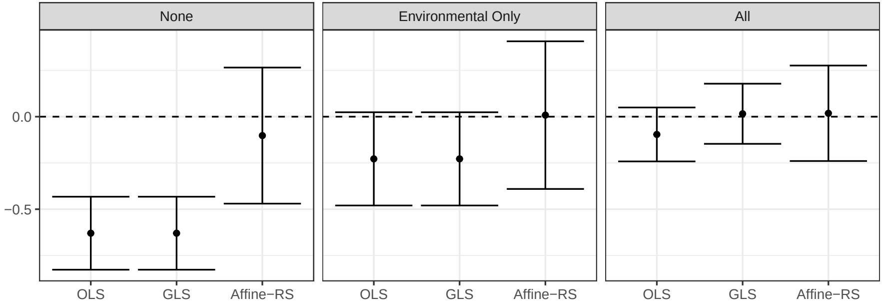
- Counties in New England in 2012
- Z: Emissions from coal power plants in the county [Henneman et al., 2019]
- Y: Average annual $\text{PM}_{2.5}$ concetration
- Covariates: Power plant characteristics, demographics, weather



Effect of coal emissions on ambient $PM_{2.5}$



Effect of relative humidity on ambient $PM_{2.5}$



Conclusions

- Unmeasured spatial confounding is not identified in the analysis of the effect of coal emissions on ambient $PM_{2.5}$ concentrations
- The affine estimator appears to mitigate unmeasured spatial bias in the analysis of the effect of relative humidity on $PM_{2.5}$

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- Unmeasured confounding is one of the main criticisms of air pollution epidemiology
- We can address the sensitivity of results through
  - Sensitivitiy analysis
  - Analysis mitigating bias by unmeasured structured confounders

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